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Non-minimally Coupled Tachyon and Inflation

Yun-Song Piao[a,c], Qing-Guo Huang[c], Xinmin Zhang[a] and Yuan-Zhong Zhang[b,c]

[a] *Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(4), Beijing 100039, China*
[b] *CCAST (World Lab.), P.O. Box 8730, Beijing 100080*
[c] *Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, China* [1]

Abstract

In this paper, we consider a model of tachyon with a non-minimal coupling to gravity and study its cosmological effects. Regarding inflation we will show that for a specific coupling of tachyon to gravity this model satisfies observations and solves various problems which exist in the single and multi tachyon inflation models.

[1] Email address: yspiao@itp.ac.cn, huangqg@itp.ac.cn

Recently there has been a lot of studies on the tachyon cosmology [1, 2, 3, 4, 5, 6]. Regarding inflation the unconventional form of the tachyonic action brings some new features and makes it different from that with a normal scalar field, however as pointed out firstly by Kofman and Linde [5] that the tachyonic inflations suffer from serious difficulties. Authors of Ref. [6] considered a model of multi tachyonic inflation, which solves some of the problems in the single tachyon model. But still for the string scale $M_s \leq H$ as required in these inflation models implies that the size of the de-Sitter horizon be smaller than the string length *i.e.* $\frac{1}{H} \leq l_s$, which makes it invalid to describe the tachyon condensation by using an effective field theory [5].

In all of these studies, the tachyon action is usually the DBI action with the minimal coupling between the tachyon and gravity [1, 7, 8]. In this paper, we propose a model of tachyon which couples to gravity non-minimally and study its cosmology. We will show that for a specific form of non-minimal coupling of the tachyon to gravity this model satisfies observations and overcomes all of the problems which exist in the single and multi tachyon inflation models. Studies in the bosonic string or for non-BPS brane indicate the possibility on the non-minimal couplings between tachyon and gravity [2][9].

The model under consideration is described by an 4D effective action of tachyon non-minimally coupled to gravity as follows

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(T)R + \int d^4x\sqrt{-g}V(T)\sqrt{1+\alpha' g^{\mu\nu}\partial_\mu T\partial_\nu T}, \tag{1}$$

where $\kappa^2 = \frac{1}{M_p^2}$ sets the 4D Planck scale and the tachyon potential around $T = 0$ is given by $V(T) = \tau_3 e^{-T^2}$. $f(T)$ is a function of tachyon field T, which corresponds to the minimal coupling of tachyon to gravity when $f(T) = 1$. The tension of the non-BPS brane is

$$\tau_3 = \frac{\sqrt{2}M_s^4}{(2\pi)^3 g_s}, \tag{2}$$

with g_s being the string coupling and $M_s = l_s^{-1} = \frac{1}{\sqrt{\alpha'}}$ the fundamental string mass and length scales. The Planck mass in this model is given by the dimensional reduction

$$M_p^2 = \frac{vM_s^2}{g_s^2}, \tag{3}$$

where $v = (M_s r)^6$, r is the radius of the compactification. The 4D effective theory is applicable only if $v \gg 1$.

Following [10], we perform a conformal transformation $g_{\mu\nu}(x) \to f(T)g_{\mu\nu}(x)$, and obtain that

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + \frac{3}{2}(\frac{f'}{f})^2 g^{\mu\nu}\partial_\mu T\partial_\nu T\right) + \int d^4x\sqrt{-g}\tilde{V}(T)\sqrt{1+\alpha' f(T)g^{\mu\nu}\partial_\mu T\partial_\nu T}, \tag{4}$$

[2] We thank A.A. Tseytlin for communication about this with us.

in Einstein frame, where $\tilde{V}(T) = \frac{V(T)}{f^2(T)}$ is the tachyon effective potential in this frame. For a spatially homogeneous but time-dependent tachyon field, the Hubble equation is given by

$$H^2 = \frac{\kappa^2}{3}\frac{\tilde{V}(T)}{\sqrt{1-\alpha' f(T)\dot{T}^2}} + \frac{1}{4}\left(\frac{f'}{f}\right)^2 \dot{T}^2, \tag{5}$$

and the equation of motion for tachyon is

$$\left(\frac{1}{1-\alpha' f(T)\dot{T}^2} + \frac{3}{2}(\frac{f'}{f})^2 \frac{M_p^2}{\alpha' f\tilde{V}}\sqrt{1-\alpha' f\dot{T}^2}\right)\ddot{T} + \left(1 + \frac{3}{2}(\frac{f'}{f})^2 \frac{M_p^2}{\alpha' f\tilde{V}}\sqrt{1-\alpha' f\dot{T}^2}\right) 3H\dot{T}$$
$$+\frac{1}{\alpha'}\frac{\tilde{V}'}{\tilde{V}f} + \frac{1}{2}\left(\frac{1}{1-\alpha' f(T)\dot{T}^2} + \frac{3M_p^2}{\alpha' f\tilde{V}}\frac{f''f - f'2}{f^2}\sqrt{1-\alpha' f\dot{T}^2}\right)\left(\frac{f'}{f}\right)\dot{T}^2 = 0. \tag{6}$$

When $f(T) = 1$, we have checked that the (5) and (6) agree with those for a minimally coupled tachyon. For $f(T) \neq 1$, however the potential is rescaled to be $\tilde{V}(T)$, besides there are more terms proportional to $\dot{T}^2$ and $\left(\frac{f'}{f}\right)^2$ in (6), which can be neglected consistently when $\delta \equiv \frac{M_p^2}{\alpha' f\tilde{V}}\left(\frac{f'}{f}\right)^2 \ll 1$.

In the following, we study the inflationary solution of this model. The slow-rolling parameters are given by

$$\epsilon = \frac{M_p^2}{\alpha'}\frac{\tilde{V}'^2}{\tilde{V}^3 f}, \tag{7}$$

$$\eta = \frac{M_p^2}{\alpha'}\frac{\tilde{V}''}{\tilde{V}^2 f}. \tag{8}$$

In the slow-rolling approximation $\epsilon, \eta \ll 1$ and $\delta \ll 1$, the Hubble equation and the equation of motion for the rolling tachyon can be expressed as

$$H^2 = \frac{\kappa^2 \tilde{V}}{3}, \tag{9}$$

$$3H\dot{T} + (\alpha'\tilde{V}f(T))^{-1}\tilde{V}' = 0. \tag{10}$$

To solve Eqs.(9) and (10) we need the specific forms of $V(T)$ and $f(T)$. Assuming that inflation starts near the top of the tachyon potential, *i.e.* around $T = 0$, we take the tachyon potential $V(T) = \tau_3 e^{-T^2}$ which valids around the maximum and has a maximum at $T = 0$. For $f(T)$ we expand it about $T = 0$ as follows

$$f(T) = 1 + \sum_i c_i T^{2i}, \tag{11}$$

where c_i are coefficients. Thus

$$\tau_3^{-1}\tilde{V}(T) = \tau_3^{-1}\frac{V(T)}{f^2(T)} = 1 - (1+2c_1)T^2 + (\frac{1}{2} + 2c_1 - 2c_2 - c_1^2)T^4 + \mathcal{O}(T^6). \tag{12}$$

Applying the slow-rolling condition $\eta < 1$, we obtain

$$g_s > \frac{(2\pi)^3}{\sqrt{2}} v \left((1 + 2c_1) + \mathcal{O}(1)T^2 + \cdots \right). \tag{13}$$

One can see that when $c_1 \neq -\frac{1}{2}$, the result is similar to that for a minimally coupled tachyon. For $c_1 = -\frac{1}{2}$, however the leading term proportional to T^2 disappears, correspondingly the "effective potential" of tachyon $\tau_3^{-1}\tilde{V}(T)$ becomes flatter. In this case, Eq.(13) becomes

$$g_s > 10^2 v T^2. \tag{14}$$

We have also checked that when $c_1 = -\frac{1}{2}$, $\delta \simeq \eta$ indeed a small parameter. This justifies our assumption above of $\delta \ll 1$ during inflation, so that we can safely drop them off in the calculation.

The amplitude of the gravitational waves produced during inflation is

$$\mathcal{P}_g \sim \frac{H}{M_p} < 3.6 \times 10^{-5}. \tag{15}$$

Substituting (9) and (3) into (15), we have

$$g_s^3 < 7 \times 10^{-7} v^2. \tag{16}$$

Then considering (14) and (16), we obtain

$$vT^6 < 10^{-13}. \tag{17}$$

One can see that for $v \gg 1$, when $T < 10^{-3}$, the condition (17) is satisfied, and in the meantime, combining (8) and (9), we have $H^{-1}T^2 < l_s$. As is seen in the following, in this model, the string length is far less than the cosmological horizon during inflation, which means that the main problem [5] troubling the single and multi tachyon inflation models is overcome.

The number of e-folds during inflation is

$$N = \int H dt \simeq - \int_{T_{60}}^{T_{end}} \frac{\alpha' f \tilde{V}(T)}{\tilde{V}'(T)} H^2 dT. \tag{18}$$

In Eq.(18) T_{60} is the field value corresponding to $N \simeq 60$ as required when the COBE scale exits the Hubble radius, and T_{end} is the field value at which inflation ends, which is determined by $\eta \sim 1$. In our model,

$$T_{60} \sim 0.1 T_{end} \sim \frac{1}{10^2}\sqrt{\frac{g_s}{v}}. \tag{19}$$

Following the definition of the amplitudes of the density perturbation in Ref [11], we have

$$\mathcal{P}_s \sim \frac{H^2}{\sqrt{\alpha' \tilde{V} f(T)} \dot{T}}. \tag{20}$$

Thus

$$\mathcal{P}_s \sim \frac{g_s^2}{10^2 v^{\frac{3}{2}} T_{60}^3} \sim 10^3 g_s^{\frac{1}{2}}. \tag{21}$$

Therefore for $\mathcal{P}_s \sim 10^{-5}$, we find that $g_s \sim 10^{-16}$ and $M_s \sim 10^3$Gev, $H^{-1} \sim 10^{26} M_p^{-1} \gg l_s \sim 10^{16} M_p^{-1}$ during inflation. In this case, the string scale is about Tev.

Before concluding we should point out that the inflationary solution of Eq.(4) can also be solved by firstly expanding $\sqrt{1 - \alpha' f(T) \dot{T}^2}$ in terms of $\dot{T}^2$, then renormalizing the kinetic energy term by a re-definition of the tachyon field. With this approach we obtain the same results as those present above.

In summary, we have considered in this paper a model of tachyon which couples to gravity non-minimally and studied its cosmological effects. For a specific coupling of tachyon to gravity $f(T) = 1 - \frac{1}{2}T^2$ around $T = 0$ we have studies the inflationary solution, and found that it makes the tachyon potential flatter, an successful inflation is realized. The string length in our model is far less than the cosmological horizon during inflation, which makes it feasible to describe the tachyon condensation by using an effective field theory, and thus the main problem troubling the single and multi tachyon inflation models is overcome. Finally, we would like to mention that the usual reheating mechanism is not feasible [5] since the tachyon does not oscillate during the decay of non-BPS branes. There are some discussions about the tachyon reheating [12]. But recently, some studies have pointed out that as the tachyon evolves into the late-time, the strength of coupling to the closed string increase [13]. These results motivate us to expect that the tachyon could emit closed string radiation [14, 15], such as graviton and dilaton, into the bulk and eventually settles in the finite minimum.

This work implies that the inflation and cosmological applications of tachyon with non-minimal couplings to gravity may have more fruitful phenomena, which is worth studying further.

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